JOURNAL OF AIRCRAFT
Vol. 42, No. 5, September–October 2005

Flutter-Margin Method Accounting for Modal Parameter Uncertainties

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The flutter margin method developed by Zimmerman and Weissenburger in 1964 is a useful tool to extrapolate the flutter speed from in-flight measured frequency and decay data at preflutter airspeeds. The usefulness of the method is based on reliable and accurate modal parameter estimations, which, in reality, can exhibit a nonnegligible amount of scatter. In this paper, Zimmerman and Weissenburger's method is extended to capture more effectively the inherent uncertainty present in the test data. Two approaches are proposed such that all "good" measured values of the modal parameters are used to construct either flutter margin or flutter speed histograms. Probability-density-function (PDF) models are also developed. It is found that the flutter margin and flutter speed pdfs are not symmetrical and right skewed with mean > median > mode. Furthermore, the flutter margin follows a gamma distribution. Different flutter margin and flutter speed statistics are investigated. Related to the flutter-margin pdf, the mode is considered to be the most relevant estimate of the flutter margin; use of the mode as the appropriate central statistic addresses the uncertainty by giving a more conservative flutter-margin estimate. With regard to the pdf of the flutter speed, consistent and conservative flutter speed predictions are obtained for any of the chosen central statistics. The two approaches are evaluated on the basis of real F-18 flight data and of simulated data.

Nomenclature

$\bar{A}$ = mean of A
$\breve{A}$ = median of A
$\tilde{A}$ = mode of A
F = flutter margin
q = dynamic pressure
U_f = flutter speed
U_{fF} = flutter speed based on mean modal parameters
$U_{f\bar{F}}$ = flutter speed based on mean flutter margin
$U_{f\breve{F}}$ = flutter speed based on median of flutter margin
$U_{f\tilde{F}}$ = flutter speed based on mode of flutter margin
β = decay rate or real part of eigenvalue, $-\varsigma\omega_n$ (sec^{-1})
σ^2 = variance
ω = modal (eigen)frequency (rad/s)

I. Introduction

FLUTTER flight testing relies on the in-flight measurement of the aircraft modal characteristics at a number of preflutter airspeeds. To that effect, a test aircraft is instrumented with a range of sensors located at key locations to capture the aeroelastic modes that participate in the structural dynamics of the aircraft, namely, in the flutter mechanism. For instance, Fig. 1 illustrates an F-18 instrumented for flutter flight testing. At each test point, the aircraft is excited, and the aeroelastic response is recorded.

Received 21 January 2004; revision received 9 August 2004; accepted for publication 10 August 2004.

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Not only the most common but also the oldest technique used to analyze the response is to monitor the damping. The damping is plotted as a function of airspeed (or dynamic pressure), and the flutter speed is then extrapolated to the point where the damping is zero. A major problem with this technique is its sensitivity to scatter in the measured data coupled with the nongradual degradation of damping. The problem is magnified by the inherently large error present in damping measurement and estimation.

In 1964, Zimmerman and Weissenburger[1] proposed another extrapolation technique based on the so-called flutter margin, given by Eq. (1).

$$F = \left[\left(\frac{\omega_2^2-\omega_1^2}{2}\right)+\left(\frac{\beta_2^2-\beta_1^2}{2}\right)\right]^2 + 4\beta_1\beta_2\left[\left(\frac{\omega_2^2+\omega_1^2}{2}\right)+2\left(\frac{\beta_2+\beta_1}{2}\right)^2\right] - \left[\left(\frac{\beta_2-\beta_1}{\beta_2+\beta_1}\right)\left(\frac{\omega_2^2-\omega_1^2}{2}\right)+2\left(\frac{\beta_2+\beta_1}{2}\right)^2\right]^2 \tag{1}$$

The flutter margin is a measure of the stability and is strictly applicable only to the problem of coalescence flutter as opposed to stall flutter. It is a function of both the damping and frequencies of the mode that coalesce to cause flutter. The detailed derivation of Eq. (1) is given in Ref. 1. It is based on the typical section two-degree-of-freedom (DOF) bending-torsion idealization with quasi-steady aerodynamics and no structural damping. Once the flutter margin is calculated at each test airspeed, it is plotted as a function of dynamic pressure. The extrapolation to the point where the flutter margin is zero gives the flutter speed. One advantage of this technique over the damping based one is that it considers the value of the two modal frequencies that coalesce thus, providing a more fundamental and direct tracking of the classical flutter mechanism. Furthermore, as was suggested in Zimmerman and Weissenburger's paper,[1] and analyzed further by subsequent investigations,[2–4] the flutter margin is more dependent upon the frequencies than on the decay rates. This is consistent with the nature of this flutter mechanism and is

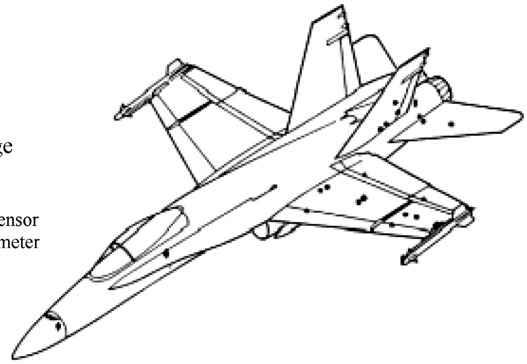

Fig. 1 F-18 instrumented for flutter flight testing; both sides instrumented.

practically significant because experimental errors in frequency are normally much smaller than errors in damping. Another advantage of the method is that the evolution of the flutter margin with dynamic pressure is much more gradual than is typically observed with damping and follows a quadratic as given by Eq. (2):

$$F = B_2 q^2 + B_1 q + B_0 \tag{2}$$

The equation parameters B_0, B_1, and B_2 are determined using a curve-fitting procedure. Even though this method offers advantages over the damping-based technique, experience has shown that it is, nevertheless, sensitive to the accuracy of the experimental data. One approach to try to enhance the accuracy is to estimate each of the four modal parameters from more than one sensor and more than one excitation event and using different parameter estimation techniques.

In this process, those data sets that are satisfactory from the perspective of yielding good quality estimates for frequency and damping for one or multiple modes will have these estimates retained. An example of this process available in the public domain is given by Katz et al.,[5] who describe a F-15 flight flutter test program. They define three categories of mode, namely, good modes, other physical modes, and fictitious modes. The good modes refer to the ones judged to give good estimates. The other physical modes are also real modes, but their modal estimates are questionable such that they are better evaluated using other sensors. For instance, a sensor located close to a nodal line of some particular mode could have a small signal-to-noise ratio. The fictitious modes have no apparent physical basis and are also discarded. However, in our opinion some of these so-called fictitious modes might in fact indicate a nonlinear aeroelastic response. These modes can provide valuable information and should be used to evaluate the inherent assumption of linearity on which most parameter identification techniques are based.[6]

The uncertainty in the modal parameters can be attributed to a number of factors such as location of the sensor, measurement noise in the sensors, air turbulence, and analysis method. Exhaustive descriptions of the different sources of uncertainties encountered in flight vibration testing are given by Koenig.[7,8] In 1984, Koenig[7] noted that, even after a careful analysis of all available data, final scatter values (defined by the coefficient of variation) of up to 2% for frequencies and 35% in damping were obtained on the good estimates. Ten years later,[8] not much improvement was reported. For instance, a comparison of 16 different analysis methods is presented showing a scatter of approximately 3% in frequency and 30% in damping.

For the conventional application of the flutter-margin method, some type of averaging, based on judgement and experience, of the good estimates is performed to come up with the best estimate (see Ref. 5 for instance). In other words, the good data are that which are used to come up with the average that forms the best for input into the flutter-margin equation (1). In this paper, we propose an extension of the flutter-margin method by making use of the inherent variation in the modal parameters derived from flight test. Specifically, the approach proposed uses all good estimates of the modal parameters to construct flutter margin and flutter speed histograms. It is based on the principle that behind this uncertainty lies information about the system which can efficiently and should effectively be carried through the analysis. Ultimately, the end product should be a more rational flutter speed prediction technique with appropriate confidence intervals and confidence levels and also with a likelihood of instability. This paper falls short of that end goal as it concentrates purely on flutter margin and flutter speed statistics; it is an intermediate but, nevertheless, a useful step between the conventional approach and a true probabilistic treatment of the problem.

For sake of completeness, it is worthwhile to mention that in addition to the flutter-margin method and the technique based on damping trends, which are the most commonly used, other flight-test flutter prediction methods exist. Two recent papers, describing and comparing the different methodologies, are given by Lind[9] and Dimitriadis and Cooper.[10] As defined by Lind, most methods are considered data-based approaches in the sense that they only use information obtained directly from flight test. This description of the flutter-margin method must be understood from the point of view that the model, given by Eqs. (1) and (2), is considered to be known. The unknowns are the modal parameters. In contrast, the method recently developed by Lind and Brenner[11] is considered to be model based. It is based on a nominal model of the aeroelastic system with unknown, but norm-bounded, parameters such as structural stiffness. It treats the uncertainties with some degree of sophistication. Nevertheless, it is a deterministic method that results in an estimated lower bound rather than a probability distribution of the flutter speed.

II. Method 1—Flutter-Margin PDF

A. Basic Flutter-Margin Statistics

Suppose that at flight-test airspeed U there are n_1 sets of good estimates for mode 1 and n_2 sets for mode 2, these being the two modes that participate in the flutter mechanism. Each set of modal estimates is a pair consisting of one value of frequency and one value of damping. Because we do not know which set is the "true" one, all combinations of mode 1 and mode 2 give possible values for the flutter margin. Accordingly, there are $n_1 \times n_2$ possible values for the flutter margin at this airspeed.

Consider the following example taken from an F-18 flight-test program. After an initial analysis of the raw flight-test data, it was decided that there were nine good estimates for mode 1 and seven good for mode 2. The estimates selected as being good were those for which a satisfactory multimode curve fit was obtained for 2–3 s following the cessation of the dwell excitation. All of the data used here were obtained in this manner, that is, examining the decay following dwell excitation and the identification was carried out using eigensystem realization algorithm with data correlation eigensystem realization algorithm with data correlation (ERA/DC).[12]

The good estimates are shown in Table 1 in addition to their respective mean values and variances. The number of possible values of the flutter margin is $9 \times 7 = 63$, which can be put in the form of a histogram as shown in Fig. 2.

Table 1 F-18 flutter flight-test modal data at $U = 39.8$[a]

Mode 1		Mode 2	
Frequency ω_1, rad/s	Damping β_1, sec^{-1}	Frequency ω_2, rad/s	Damping β_2, sec^{-1}
33.30	−0.4496	36.51	−0.3833
33.43	−0.5348	36.69	−0.3669
33.24	−0.2327	37.26	−0.6520
32.86	−0.7558	35.88	−0.7714
33.30	−0.5161	35.94	−0.4852
33.18	−0.7299	36.07	−0.5590
33.62	−0.3866	37.95	−0.5123
33.74	−0.5567	——	——
34.49	−0.2587	——	——

[a]Airspeed is nondimensional.

$\bar{\omega}_1 = 33.46$ $\quad\bar{\beta}_1 = -0.4912$ $\quad\bar{\omega}_2 = 36.61$ $\quad\bar{\beta}_2 = -0.5335$
$\sigma^2_{\omega_1} = 0.214$ $\quad\sigma^2_{\beta_1} = 0.033$ $\quad\sigma^2_{\omega_2} = 0.587$ $\quad\sigma^2_{\beta_2} = 0.021$

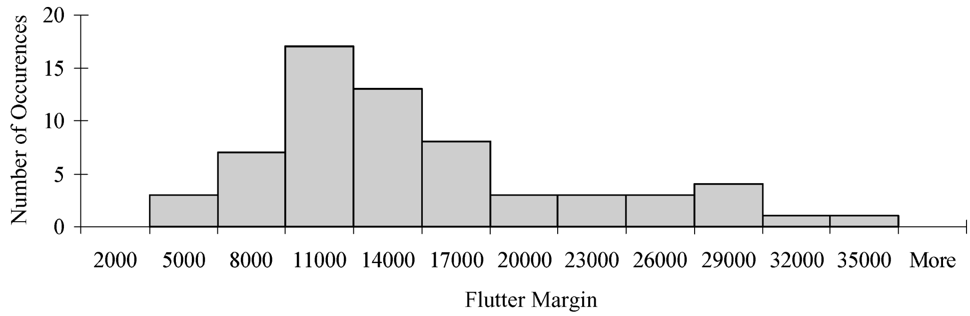

Fig. 2 Flutter-margin histogram based on Table 1 modal parameters.

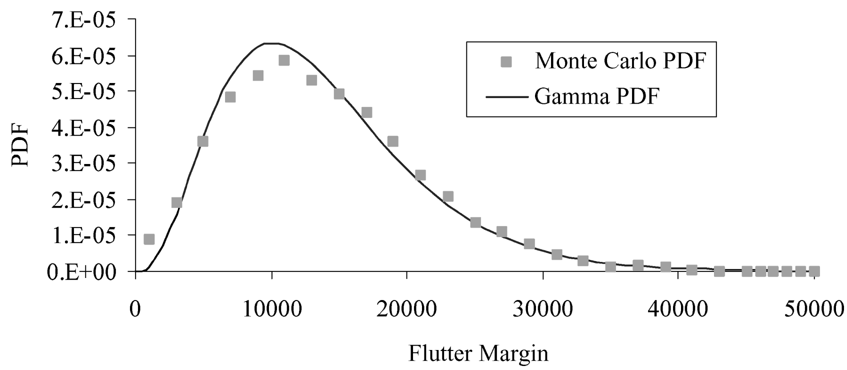

Fig. 3 Flutter-margin Monte Carlo pdf and gamma pdf, based on Table 1 modal parameters mean and variance.

The histogram is not symmetrical and is right skewed (with a long tail to the right). The information given by the histogram can be condensed in a few statistical parameters, whereby a consequence of the asymmetry is that the three measures of central tendency are different: mean $\neq$ median $\neq$ mode. Furthermore, the positive skewness leads to the following general relationship: mean > median > mode. The exact value of the mode is sensitive to the histogram bin size because the number of flutter-margin samples is statistically not very large (<100). In fact the exact shape of the histogram depends on the choice of the bin size. However, its overall characteristics are preserved. This will be discussed in further detail in the next section where a rational model of the flutter-margin pdf is discussed. Along this vein notice the hump at 29,000; it is not significant and can be attributed to one modal parameter estimate, $\omega_2 = 37.95$, which is relatively large compared to the others (see Table 1). The statistical information for this histogram is summarized as follows: mean, $\bar{F} = 13{,}796$; median, $\bar{F} = 12{,}405$; mode, $\tilde{F} = 11{,}000$; and skewness $= 0.9485$. Note that the flutter margin calculated with the mean modal parameters ($\bar{\omega}_1$, $\bar{\beta}_1$, $\bar{\omega}_2$, $\bar{\beta}_2$) given in Table 1 is: $F = 13{,}455$. As discussed earlier, it is not equal to the mean flutter margin ($\bar{F}$) shown here.

Because of the asymmetry of the histogram, there is no unique choice for the measure of the central statistics. This is significant in terms of choosing the right value of the flutter margin. It becomes a matter of judgment based on experience and risk tolerance. In this paper, we favor the choice of the mode because, by definition, it is the most probable value the flutter margin can take. In addition, it is the most conservative choice because of the positive skewness of the histogram; note that, except for one test point (airspeed) for which a negative but negligibly small skewness was observed, all experimentally derived flutter-margin histograms displayed a significant positive skewness. This observation finds some rationale in the subsequent theoretical analysis of Secs. II.B and II.C.

B. PDF Model for the Flutter-Margin Histogram

The theoretical pdf of the flutter margin, as given by Eq. (1), can be obtained by a Monte Carlo based numerical simulation. We have tried to perform an analytical development of a closed-form solution, but it proved to be intractable. The simulation is based on the assumptions that the four modal parameters are Gaussian distributed and independent random variables. These two basic assumptions are made not only because they are convenient, but they are a valid first approximation and have been used in previous studies.[2,13] They also find some physical justification in the published literature and in the fact that the uncertainty originates from different sources as described earlier. For instance, a B-2 flight flutter test program described in Ref. 14 shows a set of 23 good frequency and damping estimates. We have constructed the histograms of both the frequency and damping estimates and compared them with a Gaussian pdf. They match reasonably well, thus justifying the first assumption. We have also investigated the effect of non-Gaussian-distributed modal parameters, such as uniformly distributed, on the flutter-margin pdf. Comparing the flutter-margin distribution, mode, and mean based on uniformly distributed modal parameters with the flutter margin based on Gaussian-distributed modal parameters, we found that both cases were almost identical. We also tested the effect of the assumption of modal parameter independence by using the same seed in the random generator to simulate both modal frequencies and damping. Realizing that although this is an extreme case with correlation of one, the flutter-margin distribution appeared relatively insensitive to the correlation.

The random number generator used is RAN1 from Ref. 15. It generates uniform deviates, which are then transformed into Gaussian deviates via the Box–Muller algorithm. The example shown in Fig. 3 has been obtained using the mean and variance of the four modal parameters given in Table 1 in order to generate 4×1000 Gaussian-distributed independent random numbers. In other words, there are 1000 estimates of mode 1 (frequency and damping) and 1000 estimates of mode 2 (frequency and damping). Accordingly, the number of mode 1–mode 2 combinations is 10^6. This is a sufficiently large sample size to approximate with confidence the theoretical shape. We have found that the best model is given by the gamma distribution, whose pdf is

$$f(x) = \frac{\lambda^r x^{r-1} e^{-\lambda x}}{\Gamma(r)} \qquad \text{for} \qquad x > 0 \tag{3}$$

where Γ is the gamma function. Some relevant statistics of the gamma pdf are[16]

$$\text{Mode}: \tilde{X} = (r-1)/\lambda, \qquad \text{Mean}: \bar{X} = r/\lambda$$
$$\text{Variance}: \sigma_X^2 = r/\lambda^2 \tag{4}$$

The two parameters r and λ can be estimated with the following estimators[17]:

$$\hat{r} = \frac{\bar{X}^2}{\sum X_i^2/n - \bar{X}^2} \qquad \text{and} \qquad \hat{\lambda} = \frac{\bar{X}}{\sum X_i^2/n - \bar{X}^2} \tag{5}$$

These two estimators are based on the method of moments where, for the gamma pdf,

$$E[X] = r/\lambda \qquad \text{and} \qquad E[X^2] = r(r+1)/\lambda^2 \tag{6}$$

Using the 10^6 estimates of the flutter margin, the two estimators are calculated. They are given by $\hat{r} = 3.7516$ and $\hat{\lambda} = 0.0002722$. In turn, the mode and the mean, respectively $\tilde{F} = 10110$ and $\bar{F} = 13785$, can be calculated directly from Eq. (4). Note that using uniformly distributed modal parameters, with the same mean and variance as in Table 1, gives mode and mean, respectively $\tilde{F} = 10082$ and $\bar{F} = 13602$, which are very close to the results based on Gaussian-distributed modal parameters.

Although not perfect, the very good representation of the flutter-margin pdf by the gamma pdf can be reasoned to a certain extent by noticing that the form of the flutter margin [Eq. (1)] is defined by algebraic combinations of squared random variables. Furthermore, a special case of the gamma distribution is the widely used chi-squared distribution; the chi-squared distribution describes the summation of the square of independent unit Gaussian-distributed variables. It is not surprising that the gamma distribution provides a good model for the flutter-margin pdf. Note we also examined the log-normal distribution. The fit is also good, but the distribution parameters must be adjusted empirically. No adjustments have been made for the gamma distribution.

C. Evaluation of the Effect of Level of Scatter in Frequency and Damping

As introduced in preceding sections, the effect of variations of the modal parameters on the flutter margin has been investigated by a number of different authors.[1–4] The common conclusion is that the flutter margin is much more sensitive to variations in frequency than in damping. This is a natural consequence of the fundamental mechanism of coalescence flutter, which is largely dictated by the stiffness characteristics of the system. In this paper the effect of variations in the modal parameters is also evaluated but from the point of view of the concepts discussed in Secs. II.A and II.B.

Figure 4 presents the evolution of the mode $\tilde{F}$ of the flutter margin and its mean $\bar{F}$ as a function of the coefficient of variation; this is defined as the standard deviation divided by the mean and represents the relative level of scatter or uncertainty in the modal parameters. Three cases are shown: scatter in the first modal frequency, scatter in the second modal frequency, and scatter in all four modal parameters. The baseline deterministic conditions are the mean modal parameters ($\bar{\omega}_1$, $\bar{\beta}_1$, $\bar{\omega}_2$, $\bar{\beta}_2$) given in Table 1. Similar to the analysis of Sec. II.B, 4×1000 Gaussian-distributed independent random numbers are generated to simulate variations in the four modal parameters. The range of level of uncertainty presented represents typical values encountered in flight test; for instance, Table 1 shows a 1.4% variation in ω_1 and 2.1% in ω_2.

From Fig. 4 it is observed that, as the coefficient of variation in the modal frequencies increases, the split between the mode of the flutter margin and its mean increases significantly. Furthermore, the mode tends towards smaller values and the mean toward larger values of flutter margin. Accordingly, both the split between the flutter-margin mode and its mean, in addition to their departure from the flutter margin based on the mean modal parameters, given by $F = 13{,}455$ for this particular example, are a direct consequence of the variation in the modal parameters. More to the point, the motion of the mode towards smaller values can be considered as a measure of the uncertainty in the modal parameters. Therefore, this accounts positively for the uncertainty by making this flutter-margin estimate more conservative. This is an important conclusion.

Although not shown in this paper, we have also observed that the variance of the flutter margin, when scatter in all four parameters is considered, is approximately the addition of the variance caused by scatter in ω_1 and the variance caused by scatter in ω_2. This observation is consistent with conclusions from previous studies,[2–4] which investigated the relative effect of variations in frequency and in damping; the variation in frequency has a much greater effect. We have also analyzed the motion of the mode of the flutter margin, its mean, and its variance for the case where only damping variations are considered. For instance, even at large coefficients of variation (50%) in the damping the variance of the flutter margin is more than one order of magnitude less than the variance caused by uncertainty in frequency for a much smaller coefficient of variation (2%).

Another relevant observation is that the shape of the flutter-margin histogram, when only uncertainties in damping are considered, displays a negative skewness as opposed to a positive skewness. In other words, the mode moves towards larger values and the mean toward smaller values of flutter margin. This is the opposite behavior that is observed for the flutter-margin histogram caused by uncertainties in modal frequencies only and/or in all four modal parameters. This point also confirms the dominant effect of frequency over damping for coalescence flutter and subsequently the flutter margin.

D. Examples of Flutter Speed Calculations

In this section, we apply the approach described in the preceding sections to two different cases, and we extrapolate the flutter speed accordingly. In the first case, real data from an F-18 flutter flight-test

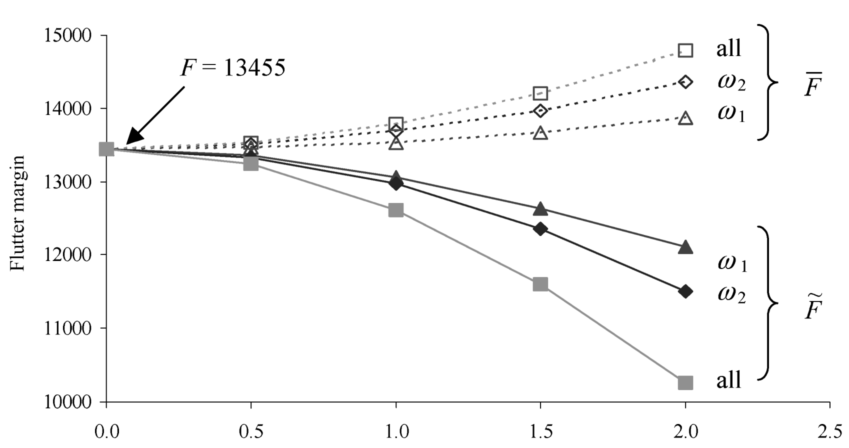

Fig. 4 Mode and mean of flutter margin as a function of coefficient of variation in ω_1, ω_2, and all parameter.

program are used. In the second case, Monte Carlo simulations of a numerical model of the archetypical two-DOF pitch-plunge elastic aerofoil are analyzed.

1. Application to F-18 Flight Flutter Test Data

Short of showing all modal parameter measured values, Table 2 describes the flight flutter test data used for this investigation. The data are then analyzed according to the histogram-based flutter margin and using the gamma pdf model as exemplified in Fig. 5 for $U = 39.8$ (airspeed is nondimensional).

The results of the flutter-margin estimation at each airspeed are summarized in Table 3 where the gamma pdf model mean $\bar{F}$ and mode $\tilde{F}$ are given, as well as the flutter margin based on the mean modal values F.

The information given in Table 3 can be used to extrapolate the flutter speed. Consequently, three flutter speed estimates are obtained based on the three types of flutter-margin estimate. At this stage of the analysis, the process is the same as for the conventional flutter-margin method, where a quadratic polynomial in dynamic pressure (or square of the equivalent airspeed) is used to extrapolate the flutter speed. This is shown in Fig. 6.

The first observation is that the mean flutter margin and the flutter margin based on the mean modal parameters are relatively close to each other, whereas the mode of the flutter margin is always clearly lower. These can be considered as general trends and are consistent with the theoretical analysis of Sec. II.C, specifically results shown in Fig. 4.

Further observations concern the flutter speed. The flutter speed extrapolation using the flutter margin based on the mean modal parameters represents the conventional approach because the variation in the test data is eliminated from the start of the analysis. In this case, the flutter speed prediction is $U_{fF} = 49.4$. All other flutter speed extrapolations are given here, including the extrapolation based on the median of the flutter margin: $U_{fF} = 49.4$; $U_{f\bar{F}} = 51.9$; $U_{f\bar{F}} = 49.9$; and $U_{f\tilde{F}} = 45.7$. The main observation is that the flutter speed predicted with the mode of the flutter margin is lower than all other predictions. This is a direct consequence of the positive skewness of the flutter-margin histogram or pdf. Taking the flutter-margin mode based extrapolation, the flutter speed is therefore predicted to be 45.7.

The fact that $U_{f\bar{F}} > U_{f\bar{F}} > U_{f\tilde{F}}$ provides some assurance that the quadratic fit flutter extrapolation is valid. This is so because this

Table 2 F-18 flutter flight-test modal data

Airspeed (nondimensional)	n_1	n_2	$\bar{\omega}_1$	$\bar{\beta}_1$	$\bar{\omega}_2$	$\bar{\beta}_2$
Ground vibration test	1	1	33.37	−0.1935	39.21	−0.3333
29.8	8	6	33.98	−0.5416	38.17	−0.4037
32.2	9	6	33.87	−0.5588	38.08	−0.362
34.8	8	6	33.62	−0.5988	37.63	−0.4233
35.9	9	6	33.78	−0.6229	37.06	−0.5128
39.8	9	7	33.46	−0.4912	36.61	−0.5335

Table 3 Results of flutter-margin estimates from F-18 flutter flight-test data

Airspeed (nondimensional)	F	$\bar{F}$	$\tilde{F}$
Ground vibration test	42,134	42,134	42,134
29.8	23,466	23,097	22,347
32.2	24,478	24,065	22,974
34.8	21,100	21,073	19,484
35.9	14,960	14,579	13,559
39.8	13,455	13,796	10,239

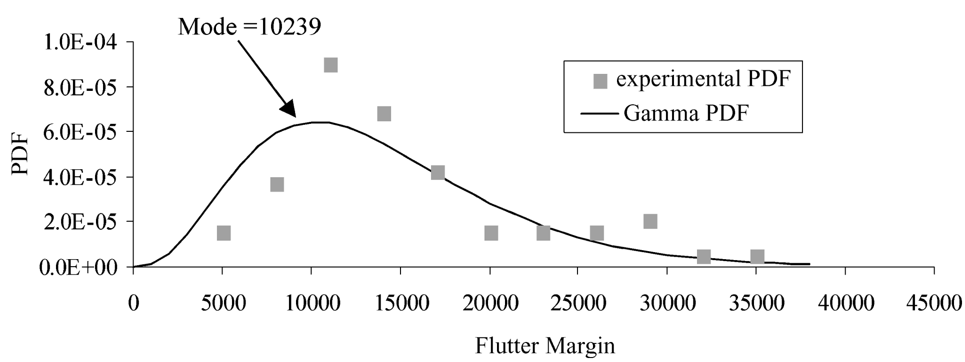

Fig. 5 Experimental flutter-margin pdf and Gamma pdf, based on Table 1 modal parameters ($U = 39.8$).

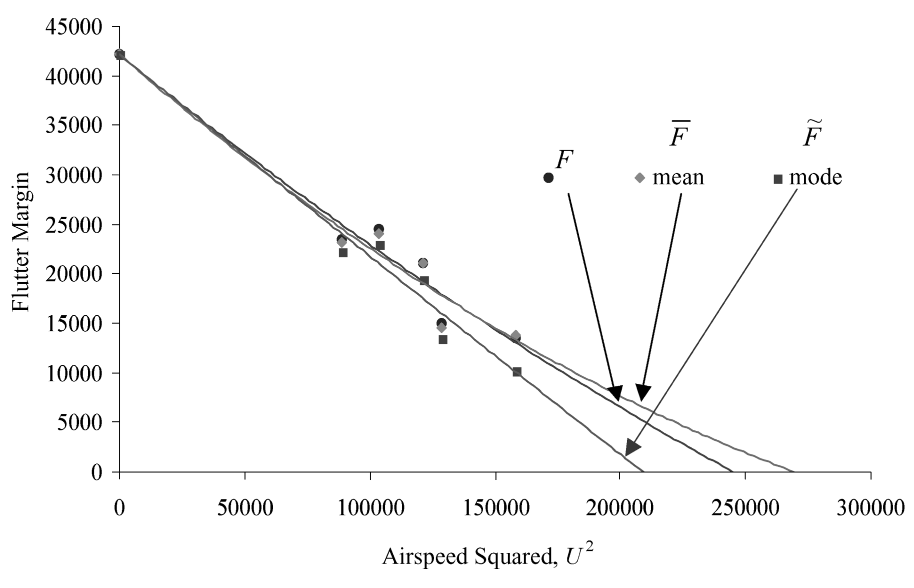

Fig. 6 Quadratic fits of flutter-margin estimates for F-18.

hierarchy in flutter speed predictions is consistent with the flutter-margin pdf model whereby mean > median > mode.

We have also investigated the effect of reducing the number of test points (airspeeds), first by removing the last test point at $U = 39.8$, and then by also removing the second last test point at $U = 35.9$. For the former case the flutter speed extrapolations were not considered valid because the $U_{f\bar{F}} > U_{f\breve{F}} > U_{f\tilde{F}}$ hierarchy was not respected. For the latter case the method predicted no flutter as the flutter margin never crossed the zero axis.

2. Application to Monte Carlo Simulation of a Two-DOF Airfoil

Applying the method to the archetypical two-DOF model enables us to compare "uncertain" flutter speed results with the known baseline deterministic flutter speed; this is not possible with real flight flutter test data because flutter is in practice rarely reached. For this example 10 simulated flight-test programs are run. Each consists of four test points (airspeed) plus the zero airspeed case. For each simulated test, 10 pairs (frequency and damping) of mode 1 and 10 pairs of mode 2 Gaussian-distributed samples are generated at the four nonzero airspeeds. Therefore, at every nonzero airspeed the number of possible values for the flutter margin is 100. The level of scatter used is based on the F-18 data given in Table 1: coefficient of variation of 1% on frequency and 30% on damping. This scatter is applied to the modal parameters at all airspeeds except at zero airspeed.

Being consistent with the physical basis of the flutter-margin model,[1] quasi-steady aerodynamics is assumed, and no structural damping is considered. For the airfoil parameters chosen the baseline flutter speed is 22.5 m/s. The chosen airspeeds and their respective deterministic modal parameters and flutter margin are given in Table 4.

For the 10 simulated experiments, we have observed a large variation in extrapolated flutter speed between each flutter-margin estimate type ranging from 21.0 to 26.3. There are even a number of cases where no flutter is predicted. These variations are simply the manifestation of the scatter in the input modal parameters and of the sensitivity of the quadratic model [Eq. (2)]. Of the 10 experiments, only one is considered to give a valid flutter speed prediction because this is only one where $U_{f\bar{F}} > U_{f\breve{F}} > U_{f\tilde{F}}$ ($U_{f\bar{F}} = 22.6$ m/s, $U_{f\breve{F}} = 21.6$ m/s, $U_{f\tilde{F}} = 21.5$ m/s). Except for the median, the quadratic fits for this experiment are shown in Fig. 7; also shown is the fit using the mean modal values F. As stated earlier, this hierarchy in flutter speed predictions is consistent with the flutter-margin pdf model whereby mean > median > mode and, thus, provides some assurance that the quadratic fit flutter extrapolation is valid. This is the criterion that we are proposing for accepting a valid flutter speed prediction. All other predictions should be rejected. Taking the flutter-margin mode-based extrapolation, the flutter speed is therefore predicted to be 21.5 m/s.

Table 4 Simulated modal parameters and flutter-margin for baseline deterministic conditions

Airspeed	ω_1	β_1	ω_2	β_2	F
0	17.24	0	105.91	0	21,481,905
10	17.59	−1.4	103.44	−0.36	17,577,058
12	17.76	−1.72	102.34	−0.39	15,561,582
15	18.07	−2.24	100.27	−0.39	11,969,243
18	18.48	−2.83	97.68	−0.33	7,839,888

The fact that only one experiment is judged to give a valid prediction might appear to be troublesome at first sight, but this is not a new problem for the flutter-margin method as discussed next. As with the conventional application of the flutter-margin method, there are two basic means of solving it. One is to use a linear model, instead of a quadratic model, to fit the flutter margin estimates in order to extrapolate the flutter speed. In theory, this approach is not substantiated, but in practice it has been used with some success, and with appropriate safety factors, in the context of the (conventional) flutter-margin method. Applied to the method described in this section, the linear fit will in most cases predict flutter speeds that follow the $U_{f\bar{F}} > U_{f\breve{F}} > U_{f\tilde{F}}$ hierarchy. However, a more rational approach is simply to increase the number of test points (speeds). Recall that this example uses only four airspeeds in addition to the zero airspeed (GVT) test point. Additional airspeeds increase the chance of having valid flutter speed predictions.

III. Method 2—Flutter Speed PDF

In this section, we extend the conceptual framework just developed by bypassing the construction of flutter-margin histograms and, instead, constructing a histogram of the flutter speed. The heart of this second method is to calculate every possible flutter speed based on all combinations of the flutter margins; a histogram of the flutter speed can then be constructed. Because the number of possible flutter speeds is very large, in this case the histogram can be interpreted as representing closely a pdf. The method is explained with the same two scenarios used in the preceding section.

A. Application to Monte Carlo Simulation of a Two-DOF Airfoil

At each airspeed there are $10 \times 10 = 100$ possible values of the flutter margin, except for the zero airspeed where we assumed there is only one. Because there are five airspeeds, $U = 0$, 10, 12, 15, and 18 m/s, the number of different combinations of the flutter margin is $1 \times 100 \times 100 \times 100 \times 100 = 10^8$. From these 10^8 combinations, 10^8 quadratic fits can be used to extrapolate flutter speeds. Similarly to the preceding application in Sec. II.D.2, 10 experiments are performed in order to assess the method. Depending on the experiment, we observe that out of these 10^8 combinations the percentage of combinations that predicts flutter ranges from 46 to 71%. The

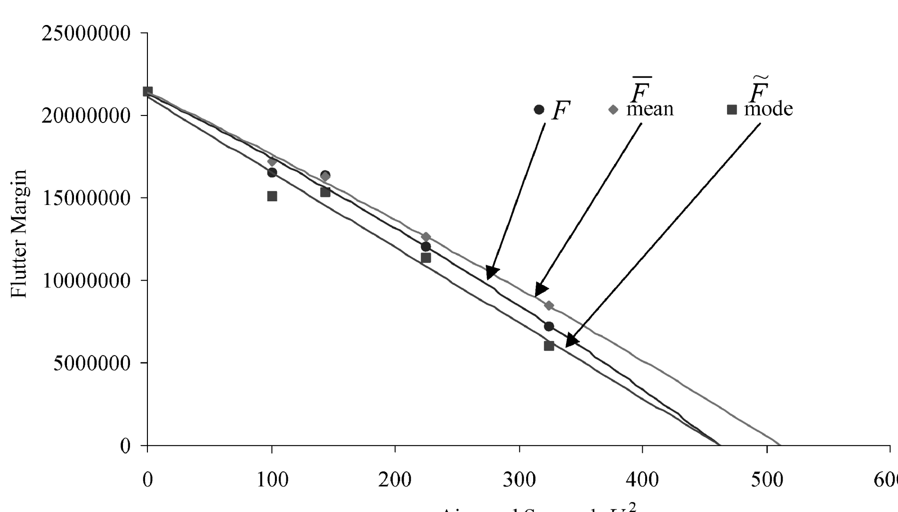

Fig. 7 Quadratic fits of flutter-margin estimates for two-DOF simulation.

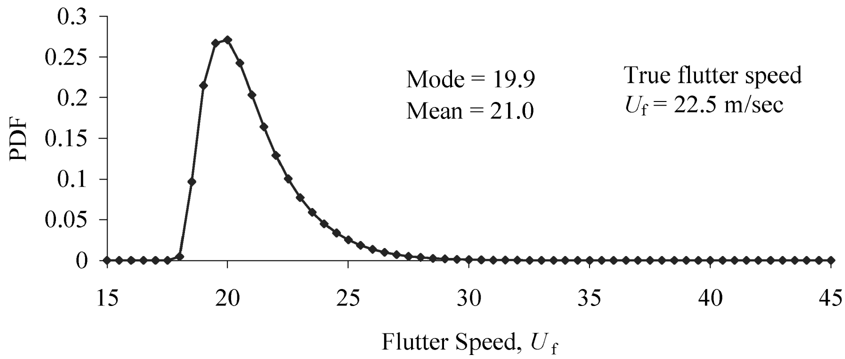

Fig. 8 Flutter speed pdf for two-DOF simulation.

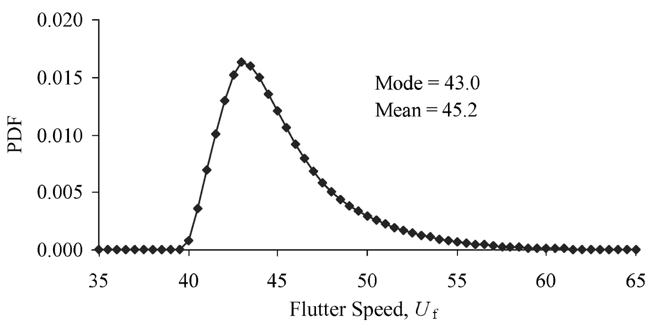

Fig. 9 Flutter speed pdf for F-18.

relatively large percentage of nonflutter predictions is as a result of the sensitivity of the quadratic fit.

The significant observation, however, is that all 10 histograms (one for each experiment) of the flutter speed are very similar qualitatively and quantitatively. They all display a positive skewness with mean > median > mode. To highlight the robustness of this second method, the experiment having the lowest percentage of flutter predictions, 46% (4.6×10^7 combinations out of 10^8), is chosen to show the flutter speed distribution. See Fig. 8 where only the combinations predicting flutter are used. It is relevant to add that, for this experiment, application of the first method described in Sec. II predicted no flutter for all flutter-margin central statistics used. The shape of the pdf shown in Fig. 8 is typical of all 10 experiments. Furthermore, the central statistics are very close for each histogram but more significantly between all 10 histograms, ranging from 19.3 to 20.5 m/s for the mode or from 20.6 to 21.6 m/s for the mean. Consistent flutter predictions are therefore obtained.

It is also relevant to point out that all three central statistics of the flutter speed are lower than the true flutter speed at $U_f = 22.5$ m/s and lower than the predicted flutter speed based on the mean modal parameters, for all 10 experiments. Additional simulations aimed at analyzing the effect of the scatter level of the modal parameters tend to demonstrate that increasing the coefficient of variation lowers the flutter speed central statistics and vice versa. Furthermore, the effect of modal parameter scatter on the flutter speed central statistics is relatively not large. It is mainly felt by the percentage of quadratic fits that do not predict flutter whereby a large scatter leads to a large number of nonflutter predictions. In all of the cases investigated, the central statistics were always below the true flutter speed and with little spread. From this point of view, this method is considered to be conservative and robust.

We have tried to find a theoretical model for the flutter speed pdf. Neither the gamma nor the log-normal model gives a good fit. In this case, contrary to the flutter margin, the development of a theoretical pdf model is not a critical issue because a very large number of experimental points are available, thus giving a smooth distribution.

B. Application to F-18 Flight Flutter Test Data

Applying this approach to the F-18 example, there are $1 \times 48 \times 54 \times 48 \times 54 \times 63 = 423{,}263{,}232$ combinations possible. Out of those combinations 60% predict flutter. The flutter speed pdf is shown in Fig. 9. It is smooth and right skewed with mean > median > mode. Its shape is very similar to the previous two-DOF example. Based on the two-DOF simulations, it is expected that the true flutter speed would not be too much above the flutter speed pdf mean, which is at 45.2. This is consistent with the predictions using the first method where the flutter speed estimate based on the flutter margin mode is 45.7.

By reducing the number of test points, we found that the percentage of flutter-margin combinations predicting flutter decreased significantly. For instance, removing the last test point at $U = 39.8$ reduces the number of combinations that predicts flutter to 44%; removing the last two airspeeds at $U = 39.8$ and 35.9 further reduces this percentage to 12%. This is also consistent with results discussed in Sec. II and stems from the sensitivity of the quadratic fit of the flutter margin. However, what is remarkable is that the flutter speed pdfs constructed from these combinations display consistent information. Their shape is right skewed with mean > median > mode. Furthermore, the mode for the case where the test point at $U = 39.8$ has been removed is at 42.0 and is at 41.0 for the other case where the two test points at $U = 39.8$ and 35.9 are not considered. Note that the mode based on all airspeeds is at 43.0.

IV. Conclusions

In principle the overall approach proposed in this paper offers a more rational treatment of the uncertainty in the modal parameters when applied to the flutter-margin method compared to the conventional application of the method. This is because the uncertainty is directly carried through the analysis resulting in the construction of flutter margin and flutter speed histograms from which different statistics are calculated.

In practice the advantages are the following. The first method proposed, based on the flutter-margin histograms, provides a criterion by which a flutter speed prediction is judged to be valid. No confidence interval is defined, but it gives some added assurance with regard to the flutter speed prediction compared with the conventional method. In addition, we found that the predicted flutter speed based on this criterion is generally conservative compared with the prediction based on the mean modal parameters, which is the essence of the conventional method. One area of improvement for this method is to develop the tools to calculate confidence intervals and levels for the three central statistics.

One drawback of the first method is the strong sensitivity of the quadratic fit of the flutter-margin estimates for the flutter speed extrapolation. However, this problem is not observed with the second method based on the construction of the flutter speed histogram. It is robust in the sense that it is relatively insensitive to the scatter in the modal parameters. It is also relatively insensitive to the number of test airspeeds, thus potentially enabling a reduction in the number of flight-test points. It is also robust from the point of view that we observed only a small spread between the three central statistics of the flutter speed histograms for all cases analyzed. Finally, it provides conservative predictions because increasing the scatter in the modal parameters tends to push the flutter speed predictions to lower values. In summary, consistent and conservative predictions are obtained without relying on having to increase unduly the number of test airspeeds nor to artificially force a linear fit of the flutter margin-dynamic pressure relationship.

The relevant question regarding which of the flutter speed central statistics is the best estimate remains open. Tied in to this question is also the calculation of confidence intervals for these statistics. Further analysis is required in this regard. One next evident step is to use real flutter test data for which the flutter speed is known, that is, using a test article that has been flight tested or tested in a wind tunnel up to the flutter speed. Nevertheless, we believe that any of the two proposed methods can be used as complements to currently accepted flight-test flutter prediction methods. No additional hardware is required, and very little software needs to be implemented.

The second method can potentially be taken a step further by tapping into the real strength of probabilistic analysis, which is to associate probability levels to flutter speeds. With appropriate confidence levels as opposed to choosing the best flutter speed estimate, there might then be scope for modifying the current safety factor approach used for flutter clearances based on flight test.

Acknowledgment

The support of the Natural Sciences and Engineering Research Council of Canada is gratefully acknowledged.

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